Levels of Inquiry Learning Sequences for Introductory Physics – Home School Instruction (by Carl J. Wenning)

Based on the Modeling Method of Instruction using Levels of Inquiry Model of Science Teaching as a framework.

UNIT 1: Scientific Thinking in Experimental Settings

Discovery Learning	Interactive Demonstration	Inquiry Lesson
Give students a string and various masses. Explain how to make a simple pendulum. Have them identify the system and system variables (things that can change and have numbers applied to them). Have students "play" with the apparatus to identify these variables. After concept is developed, apply the terms period, amplitude, mass, and length. Have the students discover as many qualitative relationships among the system variables as possible, *including gravity.* Note the need for a controlled experiment – one independent variable and one dependent variable while all other system variables are held constant – get precise information. Whiteboard and share results.	The teacher asks, "What happens to the period of the pendulum when the mass, amplitude, and length are doubled?" Have students make predictions and explain their reasoning. Students then make written predictions and both predictions and observations are tabulated and compared using a stopwatch and the appropriate experimental technique. It should become clear that T more or less independent of the mass of the bob and the amplitude of the swing (given the amount of experimental error at this level). Explain the need for a model to determine the precise relationship between period, T, and length, ℓ.	After stressing the need for a model, continue increasing the length of the pendulum and measuring periods until students begin to see a relationship between period and length (tripling and then quadrupling the original length in the previous level should be adequate). This analysis can be assisted if the teacher starts off with a pendulum with a period of 1s (a length of about 0.25m). Students should state something to the effect that period squared is proportional to length, $T^2 \propto \ell$. See the *Student Lab Handbook* by Carl Wenning for mathematical techniques.
Inquiry Lab	**Real-world Applications**	**Hypothetical Inquiry**
Teacher addresses accuracy of measurements. Students determine periods for a range of lengths. Students are informed that not less than 5 data points are required and that the independent variable's largest and smallest amounts (independent variable range) should differ by a factor of at least 10 times. Students graph data and linearize as appropriate. Data are then fit with a regression line. The teacher points out that proportionalities become equalities when a constant is included. That is, one could write $T^2 = c\ell$ or $T = k\sqrt{\ell}$.	Given the form of the relationship derived from experiment, $T^2 = c\ell$ or $T = k\sqrt{\ell}$, have students make the following calculations: (1) period for a pendulum of a given length, and (2) length of a pendulum that will have a given period. Have students determine if the formula derived experimentally can be extrapolated beyond the range of the previous data. Experimentally test both predictions for accuracy. Examine absolute and relative error.	Remind students that the acceleration due to gravity is a parameter of the pendulum system. To help them understand hypothetical inquiry, extract "g" from $T = k\sqrt{\ell}$ using dimensional analysis. That is, assume that $T = f(\ell, g)$ and show that $T = \text{const}\sqrt{\frac{\ell}{g}}$ where $k = \text{const}/\sqrt{g}$. Show that $\text{const} = 2\pi$ yielding the classical form of the relationship $T = 2\pi\sqrt{\frac{\ell}{g}}$. Test the formula by experimentally determining "g".

UNIT 2: Constant Velocity Particle Model

Discovery Learning

Using a constant motion vehicle, have students describe that motion. What does one mean by motion? What is "frame of reference"? What is the "origin"? What does one mean by "direction"? What aspects of motion are "quantifiable"? What does one mean by "speed"? What does one mean by "time"? What does one mean by "constant speed"? How does one *know* when one is seeing constant speed? Distinguish between position and distance and between speed and velocity. Introduce sign convention "right is positive; left is negative" and similar including directions such as north versus south and east versus west.

Interactive Demonstration

Tie a constant motion vehicle to a string and fix the free end of the string to a point on the ground thus allowing the vehicle to move with circular motion. Have students predict average speed and average velocity after one complete circle. After one circuit, examine the concepts of distance and displacement with respect to the origin. Distinguish between vector and scalar quantities distance versus displacement and speed versus velocity. Distinguish between average speed and average velocity.

Inquiry Lesson

Derive the relationship $x = \overline{v}t + x_0$ from a position versus time graph using the description of a straight line from algebra, $y = mx + b$. Help students realize that $\overline{s} = \frac{x - x_0}{\Delta t}$ and $\overline{v} = \frac{X - X_0}{\Delta t}$ where $x - x_0 = \Delta x \equiv \text{distance}$ and $X - X_0 = \Delta X \equiv \text{displacement}$. Discuss a variety of sample graphs helping students to understand their meanings. Examples include positive, negative, and zero slopes and intercepts.

Inquiry Lab

Using stopwatches and meter sticks to collect data, students create a position versus time graph. They manually determine the best-fit line, calculate the slope and y-intercept, and compare with the results obtained using the computer program *Graphical Analysis*. Teacher compares algebraic versus physical models to address any discrepancy with the y-intercept. Fit graph with $y = mx$ (proportional fit) versus $y = mx + b$ (linear fit) as appropriate. Students model the behavior of an object whose motion is depicted on a position versus time graph. Use graph matching if an acoustical motion detector is available. Introduce students to motion maps to characterize motion depicted on a graph. Make certain that when students are given a position versus time graph, a motion map, or an algebraic representation of constant velocity motion, they can accurately create the other two representations.

Real-world Applications

Apply the knowledge derived from experiment to real-world situations:

1. Using formula (including units) from the inquiry lesson to solve standard textbook problems that are "demonstrated" prior to solving.
2. Use a formula from the inquiry lesson to (a) determine the distance of lightning strike after seeing the flash and hearing the thunder; and (2) solve the "World War I: Finding Big Bertha" problem when time between flash and sound of the canon blast are heard from two widely separated locations.
3. Using a graphical representation, find the position where two constant motion vehicles moving toward each other with different speeds will meet in a demolition derby.

Hypothetical Inquiry

Have students develop and explain what velocity versus time graphs would look like given graphs of position versus time. Using an acoustical motion detector, compare x vs t and v vs t graphs for several samples of constant motion, both positive and negative. Have students determine whether initial position (or any position at all) is available in a v vs t graph.

UNIT 3: Uniform Acceleration Particle

Discovery Learning	Interactive Demonstration	Inquiry Lesson
Students observe a non-motorized vehicle roll down an inclined plane and characterize the motion in contrast with the motion of the constant velocity vehicle. Develop the concept of accelerated motion and distinguish initial velocity from average velocity and instantaneous velocity.	Explain to students that you are going to drop a variety of small objects from rest (coin, book, coffee filter) and will have students describe the motion. Have them predict the expected motion. Introduce the concept of friction. Exhibit the need for a model to describe friction-free accelerated motion. Have students draw the expected x-t graph. Have the students predict the nature of a v-t graph. Using a photogate and picket fence, create an x-t graph and v-t graph showing that the x-t graph is a right-opening parabola. Introduce graph forms…	Develop a position versus time graph for accelerated motion using a non-motorized vehicle starting from rest on an inclined plane. Linearize the relationship showing that $x - x_0 = \frac{1}{2}at^2$. Analyze the area under the best-fit regression line. Connect graphs to real-world motions. Practice connections between x-t, v-t, and a-t graphs. Derive relationship $v = v_0 + at$ from v-t graph. Have student use an acoustical motion detector to do graph matching. Alternatively, use Graphs and Tracks simulation program. Use motion maps for accelerated motion.
Inquiry Lab	**Real-world Applications**	**Hypothetical Inquiry**
Students analyze free fall motion using either a photogate and picket fence or (better) a ball drop-force plate lab to determine the acceleration due to gravity. That is, $g = -9.8 m/s^2$. Next, students use knowledge of the pendulum relationship $\left(T = 2\pi\sqrt{\frac{\ell}{g}}\right)$ to compare the values of g given the two methods of determination. Determine percent difference.	Conduct a set of worksheet problems relating to accelerated motion. Complete the *Mystery Moon* cooperative learning project. During this exercise, students will use the knowledge of the pendulum relationship and a variation of Newton's formulation of gravity $\left(g = \frac{GM}{r^2}\right)$ to determine the mass and name of the mystery moon.	Show graphically that the average velocity for a uniformly accelerated particle is $\bar{v} = \frac{1}{2}(v + v_0)$. Show also that for an object undergoing uniformly accelerated motion that the instantaneous velocity at the middle of an interval is equal to the average velocity over the entire interval. First, use the method of graphs to do so. Second, use the relationships $x - x_0 = \bar{v}t$ and $x - x_0 = v_0 t + \frac{1}{2}at^2$ to do so. Derive the relationship $v^2 - v_0^2 = 2a\Delta x$ from prior equations $v = v_0 + at$ and $x - x_0 = v_0 t + \frac{1}{2}at^2$.

UNIT 4A: Free Particle Model (N1)

Discovery Learning	Interactive Demonstration	Inquiry Lesson
Using a “frictionless” object (toy hovercraft, air track, flat CO_2 block on flat level surface, etc.), students examine the motion of a free particle (one in which $\Sigma F = 0$). Students should conclude that when no net forces act on the object in the horizontal direction, the object maintains constant velocity. An object at rest also remains at rest. This in essence defines Newton’s first law (N1).	Referring to Galileo’s thought experiment about inertia, demonstrate that a ball rolling down an incline speeds up and rolling up an incline slows down. Ask what happens on the level region between. Elicit the two most common misconceptions: (1) objects acquire a force, and this is what keeps them moving, and (2) in the interaction between objects, the more massive provides the greater force. Identify them as misconceptions and confront and resolve. Complete worksheet dealing with force diagrams.	The teacher holds an object above a table and then releases it. Students are asked to describe the forces acting upon the falling object. Students will respond “gravity”. Define as a long-range force emanating from Earth. Ask what factors affect this force. Ask which factors are quantifiable or readily measured. Note the force sensors can be used to measure force. Students assist in designing an experiment to determine how the force due to gravity (F_g or weight) is related to the mass of an object.
Inquiry Lab	**Real-world Applications**	**Hypothetical Inquiry**
Students conduct a lab activity with spring scale (expressed in newtons, N; the weight of a medium-sized apple is about 1N) or force sensor to determine the relationship between weight (F_g) and mass. A graph of F_g versus mass is created, and the relationship (a proportionality) finds that the gravitational force constant, g, is 9.8N/kg. (This is also known as the gravitational field strength.) The final form of the relationship is $F_g = mg$.	Given the relationship $F_g = mg$, work out solutions for several problems (finding F_g given m and vice versa) and test solutions by experiment. Note that 1N is the force required to accelerate 1kg of matter at the rate of $1 m/s^2$. Ergo, the units of N/kg are the same as those of m/s^2, and acceleration. That is, $\left(\frac{N}{kg} = \frac{kgm/s^2}{kg} = \frac{m}{s^2}\right)$. Note that the direction of force due to gravity is downward therefore $g = -9.8 m/s^2$	N/A – Include at this point the reading about force diagrams.

UNIT 4B: Free Particle Model (N3)

Discovery Learning	Interactive Demonstration	Inquiry Lesson
Students drop an object (i.e., a book or a small bock of wood) and allow it to fall to the floor. They then place the object on a tabletop and the teacher asks why it does not fall to the floor. Students prepare and whiteboard the reasons, perhaps resulting in a force diagram for the object on the tabletop and another for the object in free fall.	Teacher tells students that an object is going to be placed upon a compressible spring and/or a foam pad. They are to predict what will happen in each case. Do the same for a “bridge” in which a thin slat of wood is placed across a couple of books. Again, students predict what will happen. Note that deformation of the spring, pad, and wood are all signs of force. The compression of molecules with their mutually repulsive electron clouds act like springs applying a resistive force. Bouncing a laser beam off a mirror resting on the tabletop can show that the table deforms and acts like a spring. The beam is deflected when pressure is put upon the table because the table too is acting like a compressible object.	Students are equipped with a bathroom scale between them. When one pushes, both scales read the same amount. Next, two newton scales or force sensors are used to pull one upon the other. Regardless of whether a force is a push or a pull, the forces measured by the probes are equal and opposite.
Inquiry Lab	**Real-world Applications**	**Hypothetical Inquiry**
Conduct a Hooke’s law lab in which students compare weight of a suspended object $\left(F_g = mg\right)$ with the displacement of the spring from its equilibrium position, Δx. Determine the spring constant from the relationship for restoring force, $F_r = -k\Delta x$ noting that $F_g = -F_r$, equal and opposite forces. Show via experiment that $\Sigma F = 0$ for a suspended object at rest.	Students create free body diagrams for a variety of situations, perhaps completing a worksheet of examples. Students calculate weight and normal forces exerted by surfaces. Students use Hooke’s law to describe the operation of springs.	Students explain how in a tug-of-war game even though the forces the rope are equal in magnitude and opposite in direction, it is possible that anyone can win.

UNIT 5: Constant Force Particle Model	Discovery Learning	Interactive Demonstration	Inquiry Lesson
	Students work with a modified Atwood machine (dynamics cart and track with a weight suspended over the edge), students characterize both the system and the motion of a cart accelerated by the weight suspended over the edge. Students identify what factors might affect the acceleration of the cart. Ask them about the role of friction in such a system. How might friction be taken compensated for in such a system?	Teacher demonstrates the Atwood machine varying the mass of the weight and the mass of the cart separately. Students predict what will happen to the acceleration of the cart if the mass of the weight is increased while the mass of the cart remains constant. They predict what will happen to the acceleration of the cart of the mass of the cart is increased while holding the mass of the weight constant.	Students are asked to figure out how to measure the acceleration of a cart on a track with weight – the modified Atwood machine. Teacher uses a think aloud protocol and knowledge of accelerated motion kinematics model to find $\Delta x = \frac{1}{2}at^2$. (N.B. This assumes constant acceleration.) Students graph Δx versus $\frac{1}{2}t^2$ to find the acceleration.
	Inquiry Lab	**Real-world Applications**	**Hypothetical Inquiry**
	Following an introduction about how to conduct a controlled experiment (transferring mass from cart to weight hanger), students conduct a two-part inquiry lab to determine the relationship between a and F_{net} and between a and m. This can be done as a jigsaw project to speed the effort along. One group finds $a \propto F_{net}$ and the other group finds $a \propto \frac{1}{m}$. The teacher elicits the combined result $a \propto \frac{F_{net}}{m}$ and $a = k\frac{F_{net}}{m}$ where k = 1 if and only if F_{net} is measured in newtons where $1N \equiv 1\frac{kg \cdot m}{s^2}$. That is, $F_{net} = ma$.	Students complete a worksheet dealing with Newton's second law of motion and whiteboard the results. Students conduct an analysis of the forces acting upon human bodies during car crashes. From a knowledge of kinematics, typical distances, and changes of velocity, they find that $F_{net} = ma = m\frac{\Delta v}{\Delta t}$. Determine the accelerations in terms of g's where $1g = 9.8m/s^2$. The calculations should also extend to finding $F_{12} = -F_{21}$. This becomes a prelude to the study of momentum.	Students apply what they know about force, force diagrams, and motion to develop a model that explains friction. Using a hypothetical approach, they might well conclude that the force of friction is a function of both and object's weight and surface area. They predict the mathematical form of friction and attempt to distinguish between kinetic and static friction. Students conduct independent lab activities to find that $F_s \le \mu_s F_N$ and $F_k \le \mu_k F_N$. The first can be conducted with an increasingly inclined plane (raised to point that object slides) and the second can be conducted by dragging objects with a force sensor attached.

	Discovery Learning	Interactive Demonstration	Inquiry Lesson
Unit 6: 2-D Particle Models	Students are given small objects (ping pong balls) and told to throw them and then characterize the motion. As if either the constant or accelerated motion model applies. Have students combine horizontal (constant motion) and vertical (accelerated motion) motion maps to see if such a combination has potential for explaining the projectile motion observed.	Teacher uses two-ball projectile motion demonstration asking students what will happen when two balls are released – one with zero and one with non-zero horizontal motion.	Students conduct video analysis of projectile motion using *Logger Pro* to “separate” horizontal and vertical motions for a thrown basketball. Teacher helps students to see that horizontal and vertical motions are entirely separate. That is, motion in 2-D is a combination of two models they have learned before – accelerated and constant motion. Have students make motion maps for vertical and horizontal components of motion.
	Inquiry Lab	**Real-world Applications**	**Hypothetical Inquiry**
	Students us an inclined plane or similar to accelerate a steel ball bearing or marble to a constant speed. The ball rolls onto a level tabletop and students determine a way to reliably determine this speed. Students use this information and the height of a tabletop above the floor to predict where on the floor the ball will hit (how far from the table’s edge). Treat this as a competition.	Students complete a worksheet containing projectile motion problems. Students then, using their knowledge of constant and accelerated motion models, generate the range formula for a projectile moving over horizontal ground with an initial velocity v and an angle θ to the ground, $\Delta x = -\frac{v^2}{g}\sin 2\theta$ from $\Delta x = v_x t$ and $\Delta y = v_y t + \frac{1}{2}at^2$. Time of flight comes from second equation where $\Delta y = 0$ that implies $t = -\frac{2v_y}{g}$. Substitute into $\Delta x = v_x t$ and simplify to get range formula. Note that $v_x = v\cos\theta$ and $v_y = v\sin\theta$.	N/A

Unit 7: Energy	Discovery Learning	Interactive Demonstration	Inquiry Lesson
	Students drop a metal ball from various heights to see the effect of the impact on their hand and then a mass of clay. Teacher helps students develop the concepts of work and energy (the ability to do work). Teacher helps students understand that the volume of the depression in the clay represents an amount of energy and it is related to the amount of work required to raise the ball upward at a constant speed. That is, the more work required to raise an object such as a metal ball, the greater the amount of energy it contains (and can produce a bigger depression in clay which is proportional to the amount of energy contained in the falling ball immediately prior to impact with the clay).	A teacher drops two balls, one larger and one smaller, independently to the floor and the students watch the balls rebound. The teacher asks for an explanation about why the balls don't return to their original heights. The teacher then proposes to drop a pair of stacked balls – a smaller less massive one on top and a larger more massive one on the bottom – and ask students predict and explain will happen when they hit the floor. To the students' great surprise, the bottom ball will hardly rebound from the floor whereas the small one will be propelled most quickly upward having bounced off the lower ball. Ask students about how what they observed might be explained. Explain the need for a model to deal with these energy-related phenomena. Stretched balloon idea.	Students assist in conducting a controlled experiment so that they can find the definition of KE and discover the conservation principle, $mg\Delta y = \frac{1}{2}m\Delta v^2$ or $W = \Delta E$. Students drop one ball from various heights to get different impact speeds $\left(\Delta v = \sqrt{2g\Delta y}\right)$. Students then drop balls of different mass from same height to determine the influence of mass on the volume of the pit created which is proportional to work. Students find $KE \propto m$ and $KE \propto v^2$ or $KE \propto mv^2$. Plotting work $\left(mg\Delta y\right)$ versus mv^2 gives a slope of ½. Ergo, $KE = \frac{1}{2}mv^2$.
	Inquiry Lab	**Real-world Applications**	**Hypothetical Inquiry**
	Students conduct the Hooke's law lab a second time and again get $F = k\Delta x$. From the graph of F versus Δx they derive the concept that work $\left(F\Delta x\right)$ is equal to the area under the regression line. From this they derive $PE_e = \frac{1}{2}k\Delta y^2$ (e.g., Area = $\frac{1}{2}F\Delta y$ and because $F = k\Delta y$, Area = $\frac{1}{2}\left(k\Delta y\right)\Delta y = \frac{1}{2}k\Delta y^2$.	Students complete a variety of worksheet problems using conservation of energy in its various forms. They then devise a way to deal with the loss of energy due to friction for a sliding object using the definition $F_f = \mu F_N = \mu N$. (They probably should derive this relationship from experiment as well.) That is, $W_f = F_f\Delta x = \mu_k N\Delta x = \mu_k mg\Delta x$. They then generalize the work-energy theorem, $\Delta E = \Sigma F\Delta x = \left(F - F_f\right)\Delta x$ and apply to new situations.	Why do kinematic relationships hold and what does this have to do with energy? Consider the following kinematic equation: $v^2 - v_o^2 = 2a\Delta x$ $\frac{1}{2}m\left(v^2 - v_o^2\right) = \frac{1}{2}m\left(2a\Delta x\right) = ma\Delta x = F\Delta x$ $\Delta KE = F\Delta x = W$ Kinematics relationships hold because energy is conserved. Can other kinematics relationships be derived from this principle?

Unit 8: Central Force Particle Model

Discovery Learning	Interactive Demonstration	Inquiry Lesson
Use models of vectors to have students determine the direction of acceleration of a particle in circular motion. Find vectors v_1 and v_2 tangent to a circle. That is, $\Delta v = v_2 - v_1 = v_2 + (-v_1)$. Students find that Δv is directed toward the center.	Using a whirligig apparatus, swing a rubber stopper on a string over the head… Ask students why the weight at the base of the whirligig goes up as the speed of the rubber stopper increases. Increase the radius of the stopper and swing again. Work out the general principles that as the tangential velocity, v, increases, the force required to hold it is place also increases (e.g., F=f(v)). Similarly work out the facts that F is a f(m) and a f(r) as well. That is, $F = f(m,v,r)$.	Using dimensional analysis and the assumption that $F = f(m,v,r)$, help student to show that $F \propto \frac{mv^2}{r}$. Now, how to get v? $v = \frac{d}{t} = \frac{2\pi r}{T}$ where T (period) and r (radius of circular motion) are measurable. Assist students to design an experiments to test the assertion that $F \propto \frac{mv^2}{r}$.
Inquiry Lab	**Real-world Applications**	**Hypothetical Inquiry**
Students conduct a jigsaw lab in which three groups independently determine F versus v, F versus r, and F versus m. Students find that $F = k\frac{mv^2}{r}$ where k=1. Because F=ma, we find at $a_{centripetal} = \frac{v^2}{r}$.	Use knowledge of centripetal acceleration and Newton's second law of motion, design an appropriate amusement park rise or analyze. Calculate the acceleration of the moon in its (assumed circular) orbit given its distance and period. That is, v=d/t where $d \equiv C = 2\pi r$ and $t=T_{sidereal}$. Then, $v = \frac{d}{t} = \frac{2\pi r}{T_{sid}} = \frac{2\pi(385{,}000{,}000m)}{2{,}361{,}000s} = 1020m/s$ $a_{moon} = \frac{v^2}{r} = .00270m/s^2$; $\frac{a_{apple}}{a_{moon}} = \frac{9.8m/s^2}{0.00270m/s^2} \approx 3600$ Knowing that $\frac{a_{apple}}{a_{moon}} = 3600 = \frac{r_{moonorbit}}{r_{earth}}$, and $F_{earth} = m_{moon}a_{moon}$, etc.,	Knowing that $\frac{a_{apple}}{a_{moon}} = \frac{9.81m/s^2}{0.00271m/s^2} = 3600 = \frac{r_{moonorbit}}{r_{earth}}$, show that, $F \propto \frac{Mm}{r^2}$ and $F = G\frac{Mm}{r^2}$. Given that $F_{earth} = m_{moon}a_{moon}$, derive Kepler's third law of planetary motion. That is, $T^2 \propto r^3$. Note that the moon does not orbit Earth; rather, it orbits the center of mass of the system making r only 80/81 its given value which is measured from the center of Earth.

N.B. $T_{sidereal} = 27.321582d = 2{,}360{,}585s$

N.B. $r_{moon} = 3.445x10^8m$

Unit 9: Impulsive Force	Discovery Learning	Interactive Demonstration	Inquiry Lesson
	Collisions, explosions, pushing off one another, etc.	Tablecloth trick. Relate to Newton's first law of motion.	Start with water balloon tossing activity. Send dynamics cart into a force sensor (compression) hitting the sensor with the extended spring. Ask about the nature and duration of the forces. Help students find out that $\bar{F}\Delta t = m\Delta v$.
	Inquiry Lab	**Real-world Applications**	**Hypothetical Inquiry**
	Students find relationship between p_i and p_f using an experimental setup, and conclude that momentum is conserved; that is, $p_i=p_f$.	Basic problems worksheet. Car crash reconstruction project.	Relate to Newton's third law…